EFFECT OF HEAT REMOVAL ON NONSTEADY REGIMES OF FIBER FORMATION

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A study is made of the effect of an increase in the viscosity of a melt with cooling on the initiation and development of fiber "draw-resonance" oscillations. The effect of heat transfer on the amplitude—frequency characteristics of fibers formed under nonisothermal conditions in the stable region is also investigated.

1. A model in which the melt is considered to be a Newtonian fluid with a constant viscosity is widely used in studies of the dynamics of fiber formation. In this model, flow in the fiber is examined in a quasi-one-dimensional approximation. The inertial and capillary forces, along with gravitational forces and friction against the air, are small compared to viscous forces in the fiber and are ignored [1, 2]. The model, quite satisfactory for glass fibers, makes it possible to study loss of stability ("draw resonance") and the response of the output parameters of the fibers to various perturbations during stable formation under isothermal conditions [3-5]. The model was also used to study the effect of a change in the viscosity of the melt on nonsteady formation [6-8]. Meanwhile, the function $\mu(x)$ was assigned independently rather than determined in accordance with the solution of the heat-propagation equation. The goal of the present work is to study the effect of heat exchange with the gaseous environment on the critical (from the point of view of loss of stability) draw ratio and on the response of the fiber parameters to different sources of perturbations under conditions of stable formation.

We will examine quasi-one-dimensional equations of continuity, momentum (in a noninertial approximation), and heat propagation:

$$\frac{\partial f}{\partial t}+\frac{\partial fV}{\partial x}=0,\ f=\pi a^2,$$
$$\frac{\partial P}{\partial x}=0,\ P=3\mu f\frac{\partial V}{\partial x},\ \mu=\mu_0\exp(U/RT), \tag{1}$$
$$\rho fc\left(\frac{\partial T}{\partial t}+V\frac{\partial T}{\partial x}\right)=-2\pi a q_w.$$

The term characterizing heat conduction in the fiber was omitted from the heat-propagation equation, which is physically valid.

We will consider both forced convective heat removal and radiative heat removal. Thus, the heat flux on the surface of the fiber is equal to

$$q_w=\frac{\lambda(T-T_\infty)}{\beta_1 a}+\varepsilon c_s\left[\left(\frac{T}{100}\right)^4-\left(\frac{T_\infty}{100}\right)^4\right]. \tag{2}$$

The parameter of convective heat transfer β_1 was calculated by the integral method of boundary-layer theory in [9] for the case of laminar flow in the air boundary layer surrounding the fiber. In accordance with [9], the relation $\beta_1=\beta_1(x)$ is determined by numerical integration [together with (1)] of a system of two ordinary differential equations $\beta a' = \varphi(\beta', Va^2)$, $\beta_{1,x}=\varphi_1(\beta', \beta_1, Va^2)$. The expressions for the functions φ and φ_1 were presented in [9].

The initial and boundary conditions for Eqs. (1):

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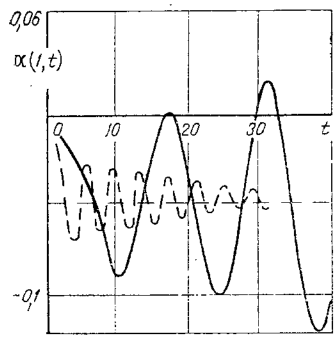

Fig. 1. Suppression of oscillations of fiber parameters due to an increase in viscosity with cooling of the melt: E = 39.6; Θ = 0 (solid line); Θ = 7 (dashed line).

$$t=0,\ a=a_*(x),\ V=V_*(x),\ T=T_*(x),$$
$$t>0\begin{cases}x=0,\ a=a_0(t),\ V=V_0(t),\ T=T_0(t),\\ x=L,\ V=V_1(t).\end{cases} \tag{3}$$

We assign the cross-sectional area, velocity, and temperature in the initial section of the fiber (hypothetically on the edge of the draw plate) and the rate of winding of the fiber on the takeup reel. The calculations were performed with the following values of the dimensionless criteria determining the solution: $R_1 = 0.207\cdot10^{-3}$, C = 1.043, $N \cong 10^5$, Re = 382.17, Pr = 0.69, $Z \cong 10^7$; also, $T_\infty/T_0 = 0.16$ (here, the value of T_0 is taken for a steady-state regime); ε = 1. The draw ratio E, determined by the steady-state values of fiber takeup and feed velocity (V_1 = const and V_0 = const), was varied in the calculation along with the dimensionless energy of activation of viscous flow Θ.

In studying the stability of formation, the stated problem was solved numerically. First we found the steady-state solution with boundary conditions which were independent of time. The nonsteady solutions were represented in the form

$$a=a_*(x)[1+\alpha(x,\ t)],\ V=V_*(x)[1+\beta(x,\ t)],$$
$$T=T_*(x)[1+\gamma(x,\ t)], \tag{4}$$
$$\alpha(x,\ 0)=\beta(x,\ 0)=\gamma(x,\ 0)=0.$$

The stationary distributions that were found were used as the initial distributions of radius, velocity, and temperature.

System (1)-(2), written relative to perturbations α, β, and γ, was solved numerically; no linearization was performed. The perturbation of the steady solution in the calculations was an instantaneous increase in takeup velocity at the moment t = 0. It turned out that cooling of the fiber, with a corresponding increase in the viscosity of the fluid, leads to expansion of the range of stable formation (Fig. 1). The results for the isothermal case (Θ = 0) are shown by the solid line – the perturbations increase and formation is unstable. The perturbations in the nonisothermal case (dashed line) decay – formation is stable, although $E > E_*$. Here $E_* = 20.22$ is the critical value at which loss of stability occurs in the isothermal regime [5].

The dashed straight line in Fig. 1 shows the new level corresponding to the steady-state solution; henceforth, the time is referred to L/V_1 and the coordinate x is referred to L (the figure corresponds to the fiber cross section on the takeup reel, x = 1). When Θ = 7, formation is stable to E ≅ 70, which agrees with the results obtained in [10] in a numerical solution of a linear eigenvalue problem. A further increase in Θ should ensure stable formation with even larger values of the draw ratio. However, the stabilizing effect connected directly with the increase in Θ may not be very great. Thus, with E ≅ 90, oscillations were seen up to Θ = 16 in the calculations. The point is that convective and radiative cooling in the model problem being examined (the length of the fiber of Newtonian fluid is not great – 10 cm) ensure a temperature drop of 30-40%. Thus, even with fairly large values of Θ, the increase in viscosity is not large enough to suppress oscillations in the outlet section of the fiber. A basic limitation on the possibility of stabilizing the oscillations is the "saturation" of the steady-state fiber configuration with an increase in Θ – the configuration nearly ceases to change. Thus, in accordance with the qualitative conclusions in [5], a further increase in Θ will not help expand the range of stable formation. Considering that, according to [11], heat removal increases sharply with transverse blowing of air over the flow, it can be concluded that the use of such measures in commercial fiber production is

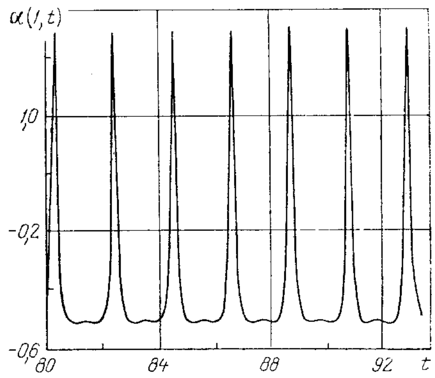

Fig. 2

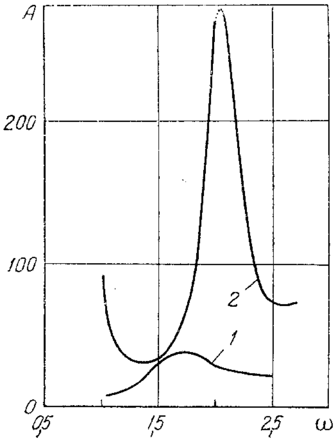

Fig. 3

Fig. 2. Oscillations of the outlet radius of the cooling fiber with a fairly large value of the draw ratio: E = 186; Θ = 7.

Fig. 3. Effect of cooling on the amplitude—frequency characteristic of the fiber; E = 16.

partly responsible for minimizing "draw resonance." The results of the calculations in [10] make it possible to evaluate the effect of transverse blowing of air on stability. We also note that while radial oscillations occur during nonisothermal formation, they are of lower amplitude than in the isothermal case for the same given draw ratio. An increase in E is initially accompanied by an increase in oscillations and a decrease in the period (opposite to the case of isothermal drawing [12]).

We also note that whereas there are no qualitative changes in the relation α(1, t) in the case of isothermal formation with an increase in E [12], in the presence of heat exchange new maxima appear between sharp peaks in this relation beginning at about E = 185 (Fig. 2; E = 186, Θ = 7). These maxima again disappear at E ≳ 336 and there is a simultaneous decrease in the amplitude of the oscillations.

2. Along with the change in the range of stability due to heat removal, there is a change in the parameters of the dynamic processes inside this region. For example, numerical solution of (1)-(3) was used in the case of stable formation to obtain the amplitude—frequency characteristic of a fiber with a radius which experienced small harmonic oscillations of the amplitude α_0 in the initial section. The amplitude of the response in the final section of the fiber is plotted off the y axis (Fig. 3)

$$A = \omega \int_0^{2\pi/\omega} [\alpha(1,\ t)]^2 dt/(\pi\alpha_0^2). \qquad (5)$$

The dimensionless frequency of the oscillations $\omega = \omega_* L/V_1$ (ω_* is the frequency) is plotted off the x axis. The value of A was determined after establishment of the oscillatory regime in the numerical solution of (1)-(3) for the case Θ ≠ 0. Curve 1 shows data obtained in the nonisothermal case for Θ = 7, while curve 2 shows the amplitude of the response in the isothermal regime (Θ = 0) calculated by means of analytical solution of linearized problem (1)-(3) [3]. The reinforcement of the oscillations decreases substantially at nearly all moderate frequencies [ω = O(1)] due to an increase in the viscosity of the fluid as a result of cooling.

It is difficult to numerically study perturbations (ω >> 1). However, the asymptotic method of multiple scales [13] can be used in the present case to obtain analytical solutions. We thus calculated the shortwave sections of the amplitude—frequency characteristics of fibers formed under nonisothermal conditions and disturbed by 1) oscillations of the inlet section of the fiber; 2) oscillations of melt feed velocity; 3) oscillations of fiber winding velocity on the takeup reel; 4) oscillations of heat flux on the fiber surface.

Here we will examine an arbitrary law of heat removal not necessarily coinciding with (2) (due to allowance for natural convection and transverse blowing of air) and we introduce the perturbation of heat removal ν:

$$q_w = q_{w*} + \nu(x, t). \tag{6}$$

Assuming α, β, γ, and ν/q_{w*} to be small compared to unity and inserting (4) and (6) into (1), after linearization we obtain the following system of dimensionless equations for the perturbations:

$$\begin{gathered}
\frac{\partial\alpha}{\partial t} + V_*\frac{\partial\alpha}{\partial x} + \frac{V_*}{2}\frac{\partial\beta}{\partial x} = 0, \\
-\frac{\Theta a_* V'_*}{T_*}\frac{\partial\gamma}{\partial x} + 2a_*V'_*\frac{\partial\alpha}{\partial x} + 2a'_*V'_*\gamma + a_*V''_*\gamma + \\
+\frac{\partial\beta}{\partial x}\left(-\frac{\Theta T'_* a_* V_*}{T_*^2} + 2a'_*V_* + 2a_*V'_*\right) + a_*V_*\frac{\partial^2\beta}{\partial x^2} = 0, \\
T_*\frac{\partial\gamma}{\partial t} + T_*V_*\frac{\partial\gamma}{\partial x} + V_*T'_*(\alpha+\beta+\gamma) = -\frac{2\nu}{a_*}.
\end{gathered} \tag{7}$$

Here the fiber radius is referred to the steady value on the takeup reel $a_1 = a_0/\sqrt{E}$, the velocity is referred to V_1, the temperature is referred to the maximum value T_0 (the temperature of the melt at the beginning of the fiber), the heat-flux perturbation is referred to $\rho c V_1 \cdot T_0 a_1/L$, the coordinate x is referred to L, and time is referred to L/V_1. The stationary distributions with respect to x denoted by an asterisk are considered to be known.

In the case of harmonic perturbations of the initial fiber radius, the boundary conditions for (7) will be

$$\alpha = \alpha_0 \sin\omega t,\ \beta = \gamma = 0,\ x = 0;\ \beta = 0,\ x = 1. \tag{8}$$

There are two length scales in the problem being examined: L and $\ell = V_1/\omega_*$ — the length of the fiber and the characteristic wavelength of the perturbation. Since $\omega = L/\ell >> 1$, the scale of the stationary distribution is much larger than the scale of the perturbation. Thus, in reality a perturbation located at a given moment near the point x evolves over distances on the order of its own characteristic dimension as if the steady-state configuration did not change along x and instead retained a given fixed value. Such a situation can be satisfactorily described by the method of multiple scales [13].

We introduce the new variables:

$$\tau = \omega t,\ x = x,\ X = \varepsilon^{-1}\int_0^x \frac{d\xi}{V_*(\xi)}. \tag{9}$$

Here the small parameter $\varepsilon = \omega^{-1} << 1$. The physical pattern described is such that the perturbations are functions of two space variables: "rapid" — X and "slow" — x, as well as of the time τ. Changing over to the new variables, we convert Eqs. (7) to the form

$$\begin{gathered}
\frac{\partial\alpha}{\partial\tau} + V_*\left(\varepsilon\frac{\partial\alpha}{\partial x} + V_*^{-1}\frac{\partial\alpha}{\partial X}\right) + \frac{V_*}{2}\left(\varepsilon\frac{\partial\beta}{\partial x} + V_*^{-1}\frac{\partial\beta}{\partial X}\right) = 0, \\
-\frac{\Theta a_* V'_*}{T_*}\left(\varepsilon\frac{\partial\gamma}{\partial x} + V_*^{-1}\frac{\partial\gamma}{\partial X}\right) + 2a_*V'_*\left(\varepsilon\frac{\partial\alpha}{\partial x} + \right. \\
\left. + V_*^{-1}\frac{\partial\alpha}{\partial X}\right) + \varepsilon(2a'_*V'_*\gamma + a_*V''_*\gamma) + \left(\varepsilon\frac{\partial\beta}{\partial x} + V_*^{-1}\frac{\partial\beta}{\partial X}\right)\times \\
\times\left(-\frac{\Theta T'_* a_* V_*}{T_*^2} + 2a'_*V_* + 2a_*V'_*\right) + a_*V_*\left(\varepsilon\frac{\partial^2\beta}{\partial x^2} - \frac{V'_*}{V_*^2}\frac{\partial\beta}{\partial X} + \frac{2}{V_*}\frac{\partial^2\beta}{\partial x\partial X} + \frac{1}{\varepsilon V_*^2}\frac{\partial^2\beta}{\partial X^2}\right) = 0, \\
T_*\frac{\partial\gamma}{\partial\tau} + T_*V_*\left(\varepsilon\frac{\partial\gamma}{\partial x} + V_*^{-1}\frac{\partial\gamma}{\partial X}\right) + \varepsilon V_*T'_*(\alpha+\beta+\gamma) = -\varepsilon\frac{2\nu}{a_*}.
\end{gathered} \tag{10}$$

All of the coefficients connected with the steady-state configuration depend only on x, while the prime denotes differentiation with respect to x. The boundary conditions (8) take the form

$$\alpha = \alpha_0 \sin\tau,\ \beta = \gamma = 0,\ x = 0;\ \beta = 0,\ x = 1. \tag{11}$$

In the case being examined, $\nu \equiv 0$.

We seek the solution of the problem in the form of asymptotic series

$$\alpha = \alpha_1 + \varepsilon\alpha_2 + \varepsilon^2\alpha_3 + \ldots,\ \beta = \beta_1 + \varepsilon\beta_2 + \varepsilon^2\beta_3 + \ldots,$$
$$\gamma = \gamma_1 + \varepsilon\gamma_2 + \varepsilon^2\gamma_3 + \ldots \tag{12}$$

Inserting (12) into (10), as a result of successive inspection of terms of the principal orders and elimination of the singularities at $X \to \infty$ and $\tau \to \infty$, we find the solution with allowance for boundary conditions (11). In particular, we obtain the solution for a perturbation of the radius of the fiber in the form

$$\alpha(x,\ t) = \alpha_0 E V_*(x) J(x) \sin\left[\omega\left(t - \int_0^x \frac{d\xi}{V_*(\xi)}\right)\right],$$
$$J(x) = \exp\left[-\frac{\Theta}{2}\int_0^x \frac{V_*'(\xi)\psi(\xi)}{T_*^2(\xi)V_*^2(\xi)}\,d\xi\right]. \tag{13}$$

Here the function ψ is the solution of the Riccati equation

$$\frac{d\psi}{dx} = \frac{\Theta V_*'(x)}{2T_*^2(x)V_*^2(x)}\psi^2(x) - T_*'(x)V_*(x) \tag{14}$$

with the condition $\psi = 0$, $x = 0$.

We did not use a specific form of the steady-state fiber configuration in constructing solution (13). Thus, (13) is also capable of describing isothermal formation if we take $\Theta = 0$, $V_* = E^{-(1-x)}$, $a_* = V_*^{-1/2}$ [2]. The result obtained here coincides with the high-frequency limit of the corresponding solution in [3]. Thus, in the isothermal case, perturbations of the fiber radius initiated in the initial section of the fiber will be reinforced E times and reach the takeup reel, where $V_* = 1$ (accordingly, $A = E^2$).

Nonisothermality of the formation process and the associated change in melt viscosity have a significant effect on the evolution of perturbations. In accordance with (13), the amplitude of fiber perturbations on the takeup reel turns out to be lower than E by the factor J(1). In calculations with (13) and (14), we can use both numerically established steady-state solutions of system (1) and the analytical steady-state solution for drawing with cooling from a draw plate that was obtained in [14] for large values of viscous-flow activation energy Θ. Calculations performed with the use of the solution in [14] at $\Theta = 7$ showed that shortwave perturbations are almost completely suppressed and $\alpha(1,\ t) \to 0 (A \to 0)$ due to the increase in melt viscosity resulting from cooling. Considering the data in Fig. 3, we see that suppression of high-frequency perturbations due to heat removal is significantly greater than for perturbations of moderate frequencies. (In the isothermal case, the response of the outlet section to perturbations of moderate and high frequencies is of the same order of magnitude.)

In the case of perturbations of melt feed velocity, the boundary conditions for (10) will be:

$$\alpha = \gamma = 0,\ \beta = \beta_0 \sin\tau,\ x = 0;\ \beta = 0,\ x = 1, \tag{15}$$

while in the case of oscillations of winding velocity on the reel

$$\alpha = \beta = \gamma = 0,\ x = 0;\ \beta = \beta_0 \sin\tau,\ x = 1. \tag{16}$$

As before, $\nu \equiv 0$.

The solution for perturbations of fiber radius in the given case has the form

$$\alpha(x,\ t) = \beta_0 \frac{V_*(x)}{2\omega}\left\{-\left(\frac{dK_i}{dx}\right)\Big|_{x=0} \exp\left(-\frac{\Theta}{2}\int_0^x \frac{V'_*\psi_1}{T_*^2 V_*^2}\, d\xi\right)\cos\left[\omega\left(t-\int_0^x \frac{d\xi}{V_*}\right)\right] + \frac{dK_i}{dx}\cos\omega t\right\}. \quad (17)$$

Here i = 1 in the case of boundary conditions (15) and i = 2 for boundary conditions (16). Meanwhile,

$$K_1(x) = 1 - \int_0^x S(\xi)\, d\xi \Big/ \int_0^1 S(\xi)\, d\xi,$$

$$K_2(x) = \int_0^x S(\xi)\, d\xi \Big/ \int_0^1 S(\xi)\, d\xi, \quad (18)$$

$$S(x) = \exp\left[\int_0^x R_2(\xi)\, d\xi\right],$$

$$R_2(x) = -V_*^{-1}(-\Theta T'_* V_*/T_*^2 + 2a'_* V_*/a_* + 2V'_*).$$

The function $\psi_1(x)$ is the solution of Eq. (14) with the condition

$$\psi = \frac{2}{E}\left(T'_* K_i \Big/ \frac{dK_i}{dx}\right)\Big|_{x=0},\ x = 0. \quad (19)$$

Solution (17) obtained for the isothermal case of course again reproduces the high-frequency limit of the corresponding solution [3]

$$\alpha(x,\ t) = \frac{\beta_0 \ln E}{2\omega(E-1)}\left\{V_* E \cos\left[\omega\left(t - \int_0^x \frac{d\xi}{V_*}\right)\right] - \cos\omega t\right\} \quad (20)$$

[for boundary conditions (15)]. The amplitude of the response of the fiber radius to the perturbations decreases with an increase in the frequency ω.

Calculations performed with (14) and (17)-(19) and the steady-state solution in [14] showed that in the nonisothermal case high-frequency perturbations are almost completely suppressed by the increase in viscosity.

Now let us turn to examination of the effect of high-frequency perturbations of heat flux on the fiber surface

$$\nu = \nu_0 \sin\tau \quad (21)$$

in the absence of perturbations of the boundary conditions for velocity, radius, and temperature. Perturbations of heat flux occur as a result of turbulence of the air flow created to intensify fiber cooling, instability of the temperature regime in the chamber, etc. The solution for perturbation of the fiber radius has the form

$$\alpha(x,\ t) = \frac{2\nu_0}{\omega\sqrt{E}} V_* \frac{\Theta}{2}\int_0^x \frac{V'_*}{T_*^2 V_*^2} \exp\left(-\int_0^\xi T'_* V_* u\, d\eta\right) d\xi \cos\left[\omega\left(t - \int_0^x \frac{d\xi}{V_*}\right)\right]. \quad (22)$$

Here the function u(x) is the solution of the equation

$$\frac{du}{dx} = T'_* V_* u^2 - \frac{\Theta V'_*}{2T_*^2 V_*^2};\ u = 0,\ x = 0. \quad (23)$$

Calculations performed with the use of (22) and (23) and the steady-state solution in [14] showed that at $E = 16$, $\Theta = 7$, and $\omega = 100$, the amplitude of the fiber-radius perturbation on the takeup reel will be roughly $\nu_0/5$. This value is significantly greater than in the previously examined cases of perturbations of radius and fiber feed and takeup velocities. However, in the present case as well, high-frequency perturbations are suppressed to a large extent due to the increase in viscosity with cooling.

Thus, in accordance with the results of the calculations and asymptotic analysis, the largest fiber defects created during formation under stable conditions should be due to external perturbations with moderate frequencies.

NOTATION

t, time; x, coordinate reckoned along the fiber axis; f, cross-sectional area of the fiber (it is assumed that the fiber has a circular cross section of radius a); V, axial velocity in the fiber; T, temperature; ρ, μ, c, density, viscosity, and specific heat of the melt; P, axial force in the fiber section; μ_0 and U, preexponential multiplier and viscous-flow activation energy; R, gas constant; q_w, heat flux in the direction of an external normal to the fiber surface; λ, thermal conductivity of air; T_∞, temperature of air at infinity; $c_S = 5.775\ J/(m^2 \cdot sec \cdot deg^4)$; ε, emissivity of the melt; β' and β_1, parameters of friction and convective heat transfer in the boundary layer on the fiber surface; V_0 and V_1, velocities of fiber feed and takeup (constant in the steady-state regime); L, fiber length (distance from the draw plate to the takeup reel); R_1 and C, ratio of the densities of heat capacities of the air and melt; A, amplitude of response; ω_*, frequency of perturbations from an external source ($\omega = \omega_* L/V_1$); α_0, β_0, ν_0, amplitudes of external perturbations of radius, velocity, and heat flux; α, β, γ, perturbations of radius, velocity, and temperature; $\tau = \omega t$ (here time is referred to L/V_1); X, "rapid" variable; $\varepsilon = \omega^{-1}$, small parameter. The scales used in calculating similitude criteria were the corresponding steady-state values of radius, velocity, and temperature; $E = V_1/V_0$, draw ratio; E_*, critical value; $N = (L/a_1)^2$, where a_1 is the steady-state value of the fiber radius on the takeup reel, equal to $a_0/\sqrt{E}$; $Re = V_1 L/\nu_1$, $Pr = \nu_1/\varkappa$, Reynolds and Prandtl numbers (ν_1 and $\varkappa$ are the kinematic viscosity and diffusivity of air); $Z = c_S T_0^3 L/(\rho c a_1 V_1)$; $\Theta = U/(RT_0)$. Indices: *, stationary distributions of radius, velocity, temperature, and heat flux on the fiber surface; 0, values of a, V, and T in the initial section of the fiber; 1, values of radius and velocity on the takeup reel.

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